Lecture 7: Search Strategies

Problem Solving as Search

• In the state space view of the world, finding a solution is finding a path through the state space.
• When we (as humans) solve a problem like the 8-puzzle we have some idea of what constitutes the next best move.
• It is hard to program this kind of approach.
• Instead we start by programming the kind of repetitive task that computers are good at.
• A brute force approach to problem solving involves exhaustively searching through the space of all possible action sequences to find one that achieves the goal.

Overview

• Last time
  – basic ideas about problem solving;
  – state space;
  – solutions as paths;
  – the notion of solution cost;
  – the importance of using the correct level of abstraction.

• Today
  – Automating search
    • Blind (uninformed, brute force) strategies.

• Learning outcome covered today:
  Identify, contrast and apply to simple examples the major search techniques that have been developed for problem-solving in AI;

Example: Romania Problem

Travel from Arad to Bucharest
**The Search Tree**

Search strategy: how do we choose which node to expand?

**Search Tree Exploration**

- The tree is built by taking the *initial* state and identifying the states that can be obtained by a single application of the *operators/actions* available.
- These new states become the *children* of the initial state in the tree.
- These new states are then examined to see if they are the *goal* state.
- If not, the process is *repeated* on the new states.
- We can formalise this description by giving an algorithm for it.
- We have different algorithms for different choices of nodes to expand.

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**Implementation: States vs. Nodes**

- A *state* is a (representation of) a physical configuration.
- A *node* is a data structure constituting part of a search tree that includes *state, parent node, action, path cost* $g(x)$, *depth*.

Expanding the tree creates new nodes, filling in the various fields and creating the corresponding states.

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**General Algorithm for Search**

```python
agenda = [initial state];
while agenda not empty do
    pick node from agenda;
    new nodes = apply operations to state;
    if goal state in new nodes then
        return solution;
    else add new nodes to agenda;
```

- Question: How to pick states for expansion?
- Two obvious strategies:
  - *depth* first search;
  - *breadth* first search.
Breadth First Search

• Start by expanding initial state - gives tree of depth 1.
• Then expand all nodes that resulted from previous step – gives tree of depth 2.
• Then expand all nodes that resulted from previous step, and so on.
• Expand nodes all at depth $n$ before going to level $n + 1$.

General Breadth First Search

/* Breadth first search */
agenda = [initial state];
while agenda not empty do
  pick node from front of agenda;
  new nodes = apply operations to state;
  if goal state in new nodes then
    return solution;
  else APPEND new nodes to END of agenda

Example: Romania BFS

Travel from Arad to Bucharest
D=0
Example: Romania BFS

Travel from Arad to Bucharest

D= 0

D= 1

Properties of Breadth First Search

• Advantage: guaranteed to reach a solution if one exists.
• Finds the shortest (cheapest) solution in terms of the number of operations applied to reach a solution.
• Disadvantage: time taken to reach solution.
  – Let \( b \) be branching factor - average number of operations that may be performed from any level.
  – If solution occurs at depth \( d \), then we will look at \( b + b^2 + b^3 + \cdots + b^d \) nodes before reaching solution - exponential.
  – The memory requirement is \( b^d \).
### Complexity

<table>
<thead>
<tr>
<th>Depth</th>
<th>Nodes</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>110</td>
<td>0.11 msec</td>
</tr>
<tr>
<td>4</td>
<td>11,110</td>
<td>11 msec</td>
</tr>
<tr>
<td>6</td>
<td>$10^6$</td>
<td>1.1 sec</td>
</tr>
<tr>
<td>8</td>
<td>$10^8$</td>
<td>2 mins</td>
</tr>
<tr>
<td>10</td>
<td>$10^{10}$</td>
<td>3 hours</td>
</tr>
<tr>
<td>12</td>
<td>$10^{12}$</td>
<td>13 days</td>
</tr>
<tr>
<td>14</td>
<td>$10^{14}$</td>
<td>3.5 years</td>
</tr>
<tr>
<td>16</td>
<td>$10^{16}$</td>
<td>350 years</td>
</tr>
</tbody>
</table>

Time for BFS assuming a branching factor of 10 and 1 million nodes expanded per second.

**Combinatorial Explosion!!**

### Depth First Search

- Start by expanding initial state.
- Pick one of nodes resulting from 1st step, and expand it.
- Pick one of nodes resulting from 2nd step, and expand it, and so on.
- Always expand **deepest node**.
- Follow one “branch” of search tree.

### General Depth First Search

/* Depth first search */
agenda = [initial state];
while agenda not empty do
  pick node from front of agenda;
  new nodes = apply operations to state;
  if goal state in new nodes then
    return solution;
  else put new nodes on FRONT of agenda;
Travel from Arad to Bucharest
Travel from Arad to Bucharest

Example: Romania DFS

Example: Romania DFS

Properties of Depth First Search

- Depth first search is guaranteed to find a solution if one exists, unless there are infinite paths.
- Solution found is not guaranteed to be the best.
- The amount of time taken is usually much less than breadth first search.
- Memory requirement is always much less than breadth first search.
- For branching factor $b$ and maximum depth of the search tree $m$, depth-first search requires the storage of only $bm$ nodes.
Exercise

• Consider a state space where the start state is number 1 and the successor function for state \( n \) returns two states, numbers \( 2n \) and \( 2n+10 \)

1) Draw the portion of the state space for the first 15 states.

2) Suppose the goal state is 38. List the order in which the nodes will be visited for both breadth first search and depth first search.

Summary: Basic Search Strategies

• Introduced:
  – Breadth-first search: complete but expensive.
  – Depth-first search: cheap but completeness not guaranteed.

• Next time
  – More advanced search strategies