Overview

• Last time
  – Intelligent agents and environments

• Today:
  – introduce problem solving and problem formulation
  – show how problems can be stated as state space search
  – show the importance and role of abstraction
  – define main performance measures for search

• Learning outcome covered today:
  Identify, contrast and apply to simple examples the major search techniques that have been developed for problem-solving in AI

Problem Solving

• What is a problem?
  A goal and a means for achieving the goal

• The goal specifies the state of affairs we want to bring about

• The means specifies the operations we can perform in an attempt to bring about the goal

• The difficulty is deciding what order to carry out the operations

• Solution will be a sequence of operations leading from initial state to goal state (plan)

Problem Formulation

More precisely, a problem can be defined by the following items:

States – a set;
  • Initial state – a particular state where the agent starts off;
  • Actions – are applicable in states;
  • Transition model – specified by a function that returns the state that results from performing action a in state s;

Goal test - determines whether a given state is a goal state (may be explicit or implicit);
  • Path cost - a function that assigns a numeric cost to each path.
  • A solution is a sequence of actions leading from the initial state to a goal state.
Example: Romania

On holiday in Romania; currently in Arad. Flight leaves tomorrow from Bucharest.

Example: Romania

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Romania: Problem Formulation

• States: various cities
• Initial state: in Arad
• Actions: drive between cities
  – e.g. In Arad: drive to Sibiu, drive to Zerind, drive to Timisoara, etc.
• Transitions: cities lead to
  – e.g. When in Arad, driving to Zerind results in being in Zerind, etc.
• Goal: be in Bucharest (- an explicit goal state)
• Path cost: Sum of step costs, i.e. distances between cities, in kilometres

Example: Vacuum World

• Consider an agent designed to vacuum clean.
• The world it inhabits has just two locations, squares A and B.
• The agent can perceive its location (which square it is in) and whether there is dirt in the square.
• The agent can choose to move left, move right, suck up the dirt, or do nothing.
Vacuum World Problem Formulation

- **States**: integer dirt and robot location
- **Initial state**: any can be designated
- **Actions**: Left, Right, Suck
- **Transition model**: actions have expected effects, though some result in no effect, e.g. sucking in a clean square
- **Goal test**: no dirt at all locations
- **Path cost**: step costs 1 per action, so path is number of steps

State Space Graph

- When taken together, the initial state, actions and transition model implicitly define the **state space** of the problem
  - This is the set of all states reachable from the initial state by any sequence of actions.
- The state space forms a directed network or graph in which the nodes are states and the links between nodes are actions.
- For the Romania example: the previous map can be interpreted as a state space graph if each road is viewed as standing for two driving actions, one in each direction.
- For the vacuum world, the state space graph is as follows...

Vacuum World State Space Graph

More Examples of Real World Problems

- Game playing
- Route finding - routing in computer networks, rail travel, air travel
- Touring and travelling - find a route between Aberdeen and Glasgow; travelling salesperson problem
- Assembly sequencing
- VLSI layout
- Robot navigation
Toy Problems: The n-Queens Problem

- This is a problem from chess.
- In the 8-queens version, place 8 queens on chess board so that no queen can be taken by another.
- A queen attacks any piece in the same row, column or diagonal.
- Has served as a useful test scenario for search algorithms.

n-Queens as a Search Problem

- **States:** Any arrangement of 0-8 queens on the board.
- **Initial state:** empty chess board.
- **Actions:** place queen in empty square.
  - Place queens anywhere
    - For the 8-queens problem $64^8$ possibilities.
  - Place queens only where they are not attacked already.
    - Around 2,000 possible sequences to check.
- **Transition model:** returns the board with a queen added to the specified square.
- **Goal test:** n queens on chess board so that none can take any other.

Toy Problems: The 8 Puzzle

- **States:** 3 × 3 grid filled with numbers 1–8 and a blank.
- **Initial state:** as shown on the left.
- **Goal test:** as shown on the right.
- **Actions:** $a_i$ - move any tile to left of empty square to right; ..... 
- **Transition model:** the states resulting from actions in a given state.
- **Path cost:** number of steps, with each step cost = 1.
8 Puzzle Search Space

Again we can map out all possible states reachable from the initial state to give the full search space.

Search Tree

The search tree shows nodes explored by a search algorithm to solve the problem. Root is the initial state: successor nodes found by applying operations (expanding nodes). Stops when goal is reached.

function TREE-SEARCH(problem, strategy) returns a solution, or failure
  initialize the search tree using the initial state of problem
  loop do
    if there are no candidates for expansion then return failure
    choose a leaf node for expansion according to strategy
    if the node contains a goal state then return the corresponding solution
    else expand the node and add the resulting nodes to the search tree
Search Strategy Performance

- A search strategy is defined by picking the order of node expansion.

- Completeness: does it always find a solution if one exists?
- Time complexity: number of nodes generated/expanded.
- Space complexity: maximum number of nodes in memory.
- Optimality: does it always find a least-cost solution?

- Time and space complexity: are measured in terms of
  - $b$: maximum branching factor of the search tree
  - $d$: depth of the least-cost solution
  - $m$: maximum depth of the state space (may be infinite)

Abstraction

- The real world is absurdly complex
  - therefore state space must be abstracted for problem solving.
- (Abstract) state = set of real states.
- (Abstract) action = complex combination of real actions
  - “Arad to Zerind” represents a complex set of possible routes, detours, rest stops, etc.
- For guaranteed realisability, any real state “in Arad” must get to some real state “in Zerind”.
- (Abstract) solution
  - a set of real paths that are solutions in the real world.
- Abstraction should be “easier” than the original problem.

Right Level of Abstraction

- Example: driving from city A to city B.
  - Some possible actions...
    - depress clutch;
    - turn steering wheel right 10 degrees;
  - inappropriate level of abstraction; too much irrelevant detail.
- Better level of abstraction:
  - follow A143 to Colchester for 4 miles;
  - turn right onto M12;
  - . . . and so on.
- Getting abstraction level right lets you focus on the specifics of the problem and combats the combinatorial explosion.

Solution Cost

- For most problems, some solutions are better than others:
  - 8 puzzle: number of moves to get to solution;
  - chess: number of moves to checkmate;
  - route planning: length of distance (or time) to travel.
- Mechanism for determining cost of solution is the path cost function.
- This is the length (cost) of the path through the state space from the initial state to the goal state.
Summary

• **Today**
  – Some search problems
  – Representing a search problem
  – Search trees
  – Evaluating search strategy performance

• **Next time**
  – Recursion in Prolog