

Bayesian Reinforcement Learning in Factored POMDPs

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ABSTRACT

Bayesian approaches provide a principled solution to the exploration-exploitation trade-off in Reinforcement Learning. Typical approaches, however, either assume a fully observable environment or scale poorly. This work introduces the Factored Bayes-Adaptive POMDP model, a framework that is able to exploit the underlying structure while learning the dynamics in partially observable systems. We also present a belief tracking method to approximate the joint posterior over state and model variables, and an adaptation of the Monte-Carlo Tree Search solution method, which together are capable of solving the underlying problem near-optimally. Our method is able to learn efficiently given a known factorization or also learn the factorization and the model parameters at the same time. We demonstrate that this approach is able to outperform current methods and tackle problems that were previously infeasible.

1 INTRODUCTION

Robust decision-making agents in any non-trivial system must reason over uncertainty in various dimensions such as action outcomes, the agent’s current state and the dynamics of the environment. The outcome and state uncertainty are elegantly captured by POMDPs [10], which enable reasoning in stochastic, partially observable environments.

The POMDP solution methods, however, assumes complete knowledge to the system dynamics, which unfortunately are often not easily available. When such a model is not available, the problem turns into a Reinforcement Learning (RL) task, where one must consider both the potential benefit of learning as well as that of exploiting current knowledge. Bayesian RL addresses this exploration-exploitation trade-off problem in a principled way by explicitly considering the uncertainty over the unknown parameters. While Model-based Bayesian RL have been applied to partially observable models [22], these approaches do not scale to problems with more than a handful of unknown parameters. Crucially, they model the dynamics of the environment in a tabular fashion which are unable to generalize over similar states and thus unable to exploit the structure of real-world applications. Earlier work for *fully observable* environments tackle this issue by representing states with features and the dynamics as graphs [21]. Their formulation for the MDP case, however, does not accommodate for environments that are either partially hidden or where the perception of the state is noisy.

In this work we introduce the Factored Bayes-Adaptive POMDP (FBA-POMDP), which captures partially observable environments with unknown dynamics, and does so by exploiting structure. Additionally we describe a solution method based on the Monte-Carlo Tree Search family and a mechanism for maintaining a belief specifically for the FBA-POMDP. We show the favourable theoretical guarantees of this approach and demonstrate empirically that it outperforms the current state-of-the-art methods. In particular, our

method outperforms previous work on 3 domains, of which one is too large to be tackled by solution methods based on the tabular BA-POMDP.

2 BACKGROUND

We first provide a summary of the background literature. This section is divided into an introduction to the POMDP and BA-POMDP, typical solution methods for those models, and factored models.

2.1 The POMDP & BA-POMDP

The POMDP [10] is a general model for decision-making in stochastic and partially observable domains, with execution unfolding over (discrete) time steps. At each step the agent selects an action that triggers a state transition in the system, which generates some reward and observation. The observation is perceived by the agent and the next time step commences. Formally, a POMDP is described by the tuple $\langle S, A, \Omega, D, R, \gamma, h \rangle$, where S is the set of states of the environment; A is the set of actions; Ω is the set of observations; D is the ‘dynamics function’ that describes the dynamics of the system in the form of transition probabilities $D(s', o|s, a)$; R is the immediate reward function $R(s, a)$ that describes the reward of selecting a in s ; $\gamma \in (0, 1)$ is the discount factor; and h is the horizon of an episode in the system. In this description of the POMDP, D captures the probability of transitioning from state s to the next state s' and generating observation o in the process for each action a .

The goal of the agent in a POMDP is to maximize the expectation over the cumulative (discounted) reward, also called the return. The agent has no direct access to the system’s state, so it can only rely on the *action-observation history* $h_t = \langle \vec{a}, \vec{o} \rangle_0^t$ up to the current step t . It can use this history to maintain a probability distribution over the state, also called a belief, $b(s)$. A solution to a POMDP is then a mapping from a belief b to an action a , which is called a *policy* $\pi = p(a|b)$. Solution methods aim to find an optimal policy, a mapping from a belief to an action with the highest possible expected return.

The POMDP allows solution methods to compute the optimal policy given a complete description of the dynamics of the domain. In many real world applications such a description is not readily available. The BA-POMDP [22] is a model-based Bayesian Reinforcement Learning framework to model applications where those are hard to get, allowing the agent to directly reason about its uncertain over the POMDP model. Conceptually, if one observed both the states and observations, then we could count the number of the occurrences of all $\langle s, a, s', o \rangle$ transitions and store those in χ , where

¹This formulation generalizes the typical formulation with separate transition T and observation functions O : $D = \langle T, O \rangle$. In our experiments, we do employ this typical factorization.

we write $\chi_{sa}^{s'o}$ for the number of times that s, a is followed by s', o . The belief over D could then compactly be described by Dirichlet distributions, supported by the counts χ . While the agent cannot observe the states and thus has uncertainty about the actual count vector, this uncertainty can be represented using regular POMDP formalism. That is, the count vector is included as part of the hidden state of the POMDP.

Formally, the BA-POMDP is the POMDP $\langle \bar{S}, A, \Omega, \bar{D}, \bar{R}, \gamma, h \rangle$ with (hyper-) state space $\bar{S} = \langle S, X \rangle$, where X is the countably infinite space of assignments of χ . While the observation and action space remain unchanged, a state in the BA-POMDP now includes Dirichlet parameters: $\bar{s} = \langle s, \chi \rangle$. The reward function still only depends on the underlying POMDP state: $\bar{R}(\bar{s}, a) = R(s, a)$. The dynamics of the BA-POMDP, $\bar{D} = p(s', \chi, o|s, \chi, a)$, factorize to $p(s', o|s, \chi, a)p(\chi'|s, \chi, a, s', o)$, where $p(s', o|s, \chi, a)$ corresponds to the expectation of $D_{sa}^{s'o}$ according to χ :

$$p(s', o|s, \chi, a) = P_\chi(s', o|s, a) = \frac{\chi_{sa}^{s'o}}{\sum_{s'o} \chi_{sa}^{s'o}} \quad (1)$$

If we let $\delta_{sa}^{s'o}$ denote a vector of the length of χ containing all zeros except for the position corresponding to $\langle s, a, s', o \rangle$ (where it is 1), and if we let $\mathbb{I}_a(b)$ denote the Kronecker delta that indicates (is 1 when) $a = b$, then we denote $\mathcal{U}(\chi, s, a, s', o) = \chi + \delta_{sa}^{s'o}$ and can write $p(\chi|s, \chi, a, s', o)$ as $\mathbb{I}_{\chi'}(\mathcal{U}(\chi, s, a, s', o))$. Lastly, just like any Bayesian method, the BA-POMDP requires a prior \bar{b}_0 , the initial belief over the domain state and dynamics. Typically the prior information about D can be described with a single set of counts χ_0 , and \bar{b}_0 reduces to the joint distribution $b_0(s) \times \chi_0$ where $b_0(s)$ is the distribution over the initial state of the underlying POMDP.

2.2 Learning by Planning in BA-POMDPs

The countably infinite state space of the Bayes-Adaptive model poses a challenge to offline solution methods due to the curse of dimensionality. Partially Observable Monte-Carlo Planning (POMCP)[23], a Monte-Carlo Tree Search (MCTS) based algorithm, does not suffer from this curse as its complexity is independent of the state space. As a result, the extension to the Bayes-Adaptive case, BA-POMCP [11], has shown promising results.

POMCP incrementally constructs a look-ahead action-observation tree using Monte-Carlo simulations of the POMDP. The nodes in this tree contain statistics such as the number of times each node has been visited and the average (discounted) return that follows. Each simulation starts by sampling a state from the belief, and traverses the tree by picking an actions according to UCB [1] and simulating observations according to the POMDP model. Upon reaching a leaf-node, the tree is extended with a node for that particular history and generates an estimate of the expected utility of the node. The algorithm then propagates the accumulated reward back up into the tree and updates the statistics in each visited node. The action selection terminates by picking the action at the root of the tree that has the highest average return.

The key modifications of the application POMCP to BA-POMDPs are two-fold: (1) a simulation starts by sampling a hyper-state $\langle s, \chi \rangle$

at the start and (2) the simulated step follows the dynamics of the BA-POMDP (algorithm 1). During this step first the domain state transitions and an observation is generated according to χ (algorithm 1 line 2), which in turn are then used to update the counts (algorithm 1 line 3).

Algorithm 1 BA-POMCP-STEP

Input s : domain state, χ : Dirichlets over D
Input a : simulated action
Output s' : new domain state, χ' : updated χ
Output o : simulated observation

- 1: // eq. (1)
- 2: $s', o' \sim P_\chi(\cdot|s, a)$
- 3: $\chi' \leftarrow \chi + \delta_{sa}^{s'o}$
- 4: **return** s', χ', o

Given enough simulations, BA-POMCP converges to the optimal solution with respect to the belief it is sampling states from [11]. One can compute this belief naively in closed form in finite state spaces by iterating over all the possible next states using the model's dynamics [22]. This quickly becomes infeasible and is only practical for very small environments. More common approaches approximate the belief with *particle filters* [24]. There are numerous methods to update the particle filter after executing action a and receiving observation o , of which *Rejection Sampling* has traditionally been used for (BA-)POMCP. *Importance Sampling* [6] (outlined in algorithm 2), however, has been shown to be superior in terms of the chi-squared distance [4].

Algorithm 2 IMPORTANCE SAMPLING

Input K : number of particles, \bar{b} : current belief
Input a : taken action, o : real observation
Output \bar{b}' : updated belief, \mathcal{L} : update likelihood

- 1: $\bar{b}' \leftarrow \{\}$
- 2: $\mathcal{L} \leftarrow 0$
- 3: // update belief
- 4: **for** $\langle \bar{s}, w \rangle \in \bar{b}(\bar{s})$ **do**
- 5: $\bar{s}' \sim p_{\bar{D}}(\cdot|\bar{s}, a)$
- 6: $w' \leftarrow p_{\bar{D}}(o|\bar{s}, a, \bar{s}')w$
- 7: $\mathcal{L} \leftarrow \mathcal{L} + w'$
- 8: add $\langle \bar{s}, w' \rangle$ to \bar{b}'
- 9: **end for**
- 10: // resample step
- 11: $\bar{b} \leftarrow \{\}$
- 12: **for** $i \in 1 \dots K$ **do**
- 13: $\bar{s} \sim \bar{b}'(\bar{s})$
- 14: add $\langle \bar{s}, w = \frac{1}{K} \rangle$ to \bar{b}
- 15: **end for**
- 16: **return** \bar{b}, \mathcal{L}

In Importance Sampling the belief is represented by a weighted particle filter, where each particle x is associated with a weight w_x that represents its probability $p(x) = \frac{w_x}{\sum_{i=1}^K w_i}$. Importance Sampling

re-computes the new belief given an action and observation with respect to the model's dynamics $p_{\bar{D}}(\bar{b}'|\bar{b}, a, o)$ in two steps. First, each particle is updated using the transition dynamics $p_{\bar{D}}(\bar{s}'|\bar{s}, a)$, and then weighted according to the observation dynamics $p_{\bar{D}}(o|\bar{s}, a, \bar{s}')$. Note that the sum of weights of the belief after this step $\mathcal{L}^t = \sum w_i^t$ represents the likelihood of the belief update at time t . The likelihood of the entire belief given the observed history can be seen as the product of the likelihood of each update step $\mathcal{L}_{h^t} = \mathcal{L}^t \mathcal{L}_{h^{t-1}}$. In the second step, starting on algorithm 2 of algorithm 2, the belief is resampled, as is the norm in sequential Importance Sampling.

2.3 Factored Models

Just like most multivariate processes, the dynamics of the POMDP can often be represented more compactly with graphical models than by tables: conditional independence between variables leads to the reduction of the parameter space, leading to simpler and more efficient models. The Factored POMDP (F-POMDP) [3] represents the states and observations with features and the dynamics D as a collection of Bayes-Nets (BN), one for each action.

Let us denote the featured state space $S = \{S^1, \dots, S^n\}$ into n features, and observation space $\Omega = \{\Omega^1, \dots, \Omega^m\}$ into m features. Then, more formally, a BN as a dynamics model for a particular action consists of an input node for each state feature s^i and an output node for each state and observation feature s^i and o . The topology $G \in \mathcal{G}$ describes the directed edges between the nodes, of which the possible graphs in is restricted such that the input nodes s only have outgoing edges and the observation nodes o only have incoming edges. For simplicity reasons we also assume that the output state nodes s^i are independent of themselves. The *Conditional Probability Tables* (CPTs) θ describe the probability distribution over the values of the nodes given their (input) parent values $PV_G()$. The dynamics of a F-POMDP are then defined as follows

$$D(s', o|s, a) = \left[\prod_{s'_i \in s'} P_{\theta^a}(s'_i | PV_G^a(s)) \right] \left[\prod_{o_i \in o} P_{\theta^a}(o_i | PV_G^a(s')) \right],$$

Some approaches are able to exploit the factorization of F-POMDPs, which typically leads to better solution [3]. These methods, however, operate under the assumption that the dynamics are known a-priori and hence cannot be applied to applications where this is not the case.

3 BAYESIAN RL IN FACTORED POMDPS

The BA-POMDP provides a Bayesian framework for RL in POMDPs, but is unable to describe or exploit structure that many real world applications exhibit. It also scales poorly, as the number of parameters grow quadratic in the state space, $O(|S|^2|A||\Omega|)$, where only one parameter (count) is updated after each observation. Here we introduce the Factored BA-POMDP (FBA-POMDP), the Bayes-Adaptive framework for the factored POMDP, that is able to model, learn and exploit such relations in the environment.

3.1 The Factored BA-POMDP

If we consider the case where the structure G of the underlying POMDP is known a-priori, but its parameters θ are not, then it is clear that we could define a Bayes-Adaptive model where the counts describe Dirichlet distributions over the CPTs: χ_G (which we know how to maintain over time). However, this assumption is unrealistic, so we must also consider both the topology G and its parameters χ as part of the hidden parameters, in addition to the domain state s .

We define the FBA-POMDP as a POMDP with the (hyper-) state space $\bar{S} = S \times \mathcal{G} \times X$. Let us first consider its dynamics $\bar{D} = p(\langle s', G', \chi' \rangle, o | \langle s, G, \chi \rangle, a)$. This joint probability can be factored using the same standard independence assumptions made in the BA-POMDP:

$$\bar{D}(\bar{s}' = \langle s', G', \chi' \rangle, o | \bar{s}, a) = p(s', o | \langle s, G, \chi \rangle, a) \quad (2)$$

$$p(\chi' | \langle s, G, \chi \rangle, a, s', G', o) \quad (3)$$

$$p(G' | \langle s, G, \chi \rangle, a, s', o) \quad (4)$$

Term $p(s', o | \dots)$ (eq. (2)) corresponds to the expectation of $p(s', o | s)$, under the joint Dirichlet posterior χ_{G^a} over CPTs θ_{G^a} . Given the expected CPTs $\mathbf{E}(\chi_{G^a}) = \hat{\theta}_{G^a}$, that probability is described by the product governed by the topology: $P_{\hat{\theta}_{G^a}}(s', o | s) =$

$\prod_{x \in s', o} \hat{\theta}_{G^a}^{x | PV(x)}$. Throughout the rest of the paper we will refer to this probability eq. (2) with D_{χ_G} .

$p(\chi' | \dots)$ (eq. (3)) describes the update operation on the counts χ that correspond to $\langle s, a, s', o \rangle: \mathcal{U}(\chi_G, s, a, s', o)$. Note that this will update $n + m$ counts, one for each feature. Lastly, we assume the topology of G is static over time, which reduces $p(G' | \dots)$ (eq. (4)) to the Kronecker delta function $\mathbb{I}_{G'}(G)$. This leads to the following definition of the FBA-POMDP model, given tuple $\langle \bar{S}, A, \Omega, \bar{D}, \bar{R}, \gamma, h \rangle$:

- A, γ, h : Identical to the underlying POMDP.
- $\bar{R}(\bar{s}, a) = R(s, a)$ ignores the counts and reduces to the reward function of the POMDP just like in the BA-POMDP.
- $\bar{\Omega}$: $\{\Omega^0 \times \dots \times \Omega^m\}$. Set of possible observations defined by their features.
- \bar{S} : $\{S^0 \times \dots \times S^n\} \times \mathcal{G}^A \times X_{G^A}$. The cross product of the domain's factored state space and the set of possible topologies, one for each action a , and their respective Dirichlet distribution counts.
- \bar{D} : $p(\bar{s}', o | \bar{s}, a) = D_{\chi_G}(s, a, s', o) \mathbb{I}_{\chi_G'}(\mathcal{U}(\chi_G, s, a, s', o)) \mathbb{I}_G(G)$, as described above.

A prior for the FBA-POMDP is a joint distribution over the hyper-state $\bar{b}_0(\langle s, G, \chi \rangle)$. In many applications the influence relationships between features is known a-priori for large parts of the domain. For the unknown parts, one could consider a uniform distribution, or distributions that favours few edges.

3.2 Solving FBA-POMDPs

Solution methods for the FBA-POMDP face similar challenges as those for BA-POMDPs with respect to uncountable large (hyper-) state spaces as a result of the uncertainty over current state and the

dynamics. So it is only natural to turn to POMCP-based algorithms for inspiration.

BA-POMCP extends MCTS to the Bayes-Adaptive case by initiating simulations with a $\langle s, \chi \rangle$ sample (from the belief) and applying the BA-POMDP dynamics to govern the transitions (recall algorithm 1). We propose a similar POMCP extension, the FBA-POMCP, for the factored case where we sample a hyper-state $\langle s, G, \chi \rangle$ at the start of each simulation, and apply the FBA-POMDP dynamics \bar{D} to simulate steps. This is best illustrated in algorithm 3, which replaces BA-POMCP-STEP algorithm 1. During a step the sampled $\langle G, \chi \rangle$ is used to sample a transition, after which the counts associated with that transition are updated.

Algorithm 3 FBA-POMCP-STEP

Input s : domain state, G : graph topology
Input χ : Dirichlets over CPTs in G , a : simulated action
Output s' : new domain state, G' : G
Output χ' : updated χ , o : simulated observation

- 1: $s', o' \sim p_{\bar{D} \times G}(\cdot | s, a)$
- 2: // increment count of each node-parent in place
- 3: **for** each node $n \in s', o$ and its value v **do**
- 4: $\chi_{G^a}^{n,v|PV(n)} \leftarrow \chi_{G^a}^{n,v|PV(n)} + 1$
- 5: **end for**
- 6: $s \leftarrow s'$
- 7: **return** $\langle s, G, \chi \rangle, o$

3.3 Belief tracking & Particle Reinvigoration

Because structures in the particles are not updated over time and due to particle degeneracy, traditional particle filter belief update schemes then to converge to a single structure, which is inconsistent with the true posterior, leading to poor performance. To tackle this issue, we propose a MCMC-based sampling scheme to occasionally reinvigorate the belief with new particles according to the (observed) history $p(\langle s, G, \chi \rangle | \langle \vec{a}, \vec{o} \rangle, \vec{b}_0)$.

First we introduction the notation $\vec{x}_{r \dots t}$ which describes the (sequence of) values of x from time step r to t of real interactions with the environment, with the special case of x_t , which corresponds to the value of x at time step t (where \vec{x} can be a sequence of states, action or observations). For brevity we also use ‘model’ and the tuple $\langle G, \chi \rangle$ interchangeably in this section, as they represent the dynamics of a POMDP. Lastly, we refer to T as the last time step in our history.

On the highest level we apply Gibbs sampling, which approximates a joint distribution by sampling variables from their conditional distribution with the remaining variables fixed: we can sample $p(x, y)$ by picking some initial x , and sampling $y \sim p(y|x)$ followed by $x \sim p(x|y)$. Here we pick $x = \vec{s}$ and $y = \langle G, \chi \rangle$ and sample alternatively a model given a state sequence and a state sequence given a model:

- i. $\vec{s} \sim p(\cdot | G, \chi, \langle \vec{a}, \vec{o} \rangle, \vec{b}_0)$
- ii. $G, \chi \sim p(\cdot | \langle \vec{s}, \vec{a}, \vec{o} \rangle, \vec{b}_0)$

Sample step (i) samples a state-sequence given the observed history $\langle \vec{a}, \vec{o} \rangle$ and current model. A very simple and naive approach is to use Rejection Sampling to sample state and observation sequences based on the action history and reject them based on observation equivalence, optionally exploiting the independence between episodes. Due to the high rejection rate, this approach is impractical for non-trivial domains. Alternatively, we model this task as sampling from a Hidden Markov Model, where the transition probabilities are determined by the model $\langle G, \chi \rangle$ and action history \vec{a} .

To sample a hidden state sequence from an HMM given some observations (also called *smoothing*) one typically use forward-backward messages to compute the conditionals probability of the hidden states efficiently [19]. We first compute the conditional $\forall t : p(s_t | \vec{a}_{1 \dots T}, \vec{o}_{1 \dots T}, G, \chi)$ with backward-messages and then sample $s_0 \dots s_T$ hierarchically in a single forward pass.

The second conditional (ii) in the Gibbs sampling scheme is from distribution $p(G, \chi | \langle \vec{s}, \vec{a}, \vec{o} \rangle, \vec{b}_0)$, which itself is split into two sample steps. We first **(a)** sample $G \sim p(\cdot | \langle \vec{s}, \vec{a}, \vec{o} \rangle, \vec{b}_0)$ using Metropolis-Hastings. This is followed by sampling a set of counts **(b)** $\chi \sim p(\cdot | \langle \vec{s}, \vec{a}, \vec{o} \rangle, G, \vec{b}_0)$, which is a deterministic function that simply takes the prior χ_0^G and counts the transitions in the history $\langle \vec{s}, \vec{a}, \vec{o} \rangle$. For the first sample step **(ii a)** $G \sim p(\cdot | \langle \vec{s}, \vec{a}, \vec{o} \rangle, \vec{b}_0)$ we start from the general Metropolis-Hastings case:

Metropolis-Hastings samples some distribution $p(x)$ using a proposal distribution $q(\tilde{x}|x)$ and testing operation. The acceptance test probability of \tilde{x} is defined as $\frac{p(\tilde{x})q(x|\tilde{x})}{p(x)q(\tilde{x}|x)}$. More specifically, given some initial value x , Metropolis-Hastings consists of:

- (1) sample $\tilde{x} \sim q(\tilde{x}|x)$
- (2) with probability $\frac{p(\tilde{x})q(x|\tilde{x})}{p(x)q(\tilde{x}|x)}$: $x \leftarrow \tilde{x}$
- (3) store x and go to (1)

Let us take $p(x)$ as $p(G | \langle \vec{s}, \vec{a}, \vec{o} \rangle, \vec{b}_0)$ and q to be domain specific and symmetrical, then we derive the following Metropolis-Hastings step for **(ii a)**:

$$\begin{aligned}
 MH\text{-STEP}_{accept} &= \frac{p(\tilde{G} | \langle \vec{s}, \vec{a}, \vec{o} \rangle, \vec{b}_0) q(\tilde{x} | x)}{p(G | \langle \vec{s}, \vec{a}, \vec{o} \rangle, \vec{b}_0) q(x | \tilde{x})} \\
 &= \frac{p(\langle \vec{s}, \vec{a}, \vec{o} \rangle, \tilde{G} | \vec{b}_0)}{p(\langle \vec{s}, \vec{a}, \vec{o} \rangle, G | \vec{b}_0)} \\
 &= \frac{p(\langle \vec{s}, \vec{a}, \vec{o} \rangle, G | \vec{b}_0)}{p(\langle \vec{s}, \vec{a}, \vec{o} \rangle, \tilde{G} | \vec{b}_0)} \\
 &= \frac{p(\langle \vec{s}, \vec{a}, \vec{o} \rangle, G | \vec{b}_0)}{p(\langle \vec{s}, \vec{a}, \vec{o} \rangle, G | \vec{b}_0)}
 \end{aligned}$$

Which leads to the likelihood ratio between the two graph structures. It has been shown that the likelihood $p(\langle \vec{s}, \vec{a}, \vec{o} \rangle, G | \vec{b}_0)$, given some mild assumptions (such as that the prior is a Dirichlet), this value is given by the BD-score metric [8]: $P(G|D) \propto P(G, D) = BD(G, D)$. Given some initial set of prior counts for G , χ_0 , and a dataset of occurrences N^{nev} of values v with parent values e for node n provided by $\langle \vec{s}, \vec{a}, \vec{o} \rangle$, then the score is computed as follows: $p(\langle \vec{s}, \vec{a}, \vec{o} \rangle, G | \vec{b}_0) = BD(G, \langle \vec{s}, \vec{a}, \vec{o} \rangle | \vec{b}_0) =$

$$\prod_n \prod_e \frac{\Gamma(\chi_0^{ne})}{\Gamma(\chi_0^{ne} + N^{ne})} \prod_v \frac{\Gamma(\chi_0^{nev} + N^{nev})}{\Gamma(\chi_0^{nev})}$$

Where we abuse notation and denote the total number of counts, $\sum_v \chi^{nev}$, as χ^{ne} (and similarly $N^{ne} = \sum_v N^{nev}$).

Given this acceptance probability, Metropolis-Hastings can sample a new set of graph structures G with corresponding counts for the CPTs χ . This particular combination of MCMC methods – Metropolis-Hastings in one of Gibbs’s conditional sampling steps – is also referred to as MH-within-Gibbs and, surprisingly, has shown to converge to the true distribution even if the Metropolis-Hastings part only consist of 1 sample per step [12, 13, 17, 20, 25].

The overall particle reinvigoration procedure, assuming some initial $\langle G, \chi \rangle$, is as follows:

- (1) sample from HMM $p(\vec{s}|\langle \vec{a}, \vec{d} \rangle, G, \chi, \vec{b}_0)$
- (2) sample from MH: $p(G|\langle \vec{s}, \vec{a}, \vec{d} \rangle, \vec{b}_0)$ (using BD-scores)
- (3) compute counts: $p(\chi|\langle \vec{s}, \vec{a}, \vec{d} \rangle, G, \vec{b}_0)$
- (4) add $\langle s, G, \chi \rangle$ to belief and go to 1

It is not necessary to do this operation at every time step, instead the log-likelihood \mathcal{L} of the current belief is a useful metric to determine when to resample. Fortunately, it is a by-product of Importance Sampling at line 6 of algorithm 2. The total accumulated weight, denoted as $\eta = \sum w^i$ (the normalization constant) is the likelihood of the belief update. Starting with $\mathcal{L}=0$ at $t=0$, we maintain the likelihood over time $\mathcal{L} = \mathcal{L} + \log \eta_t$ and update the posterior $b(\langle s, G, \chi \rangle|\langle \vec{a}, \vec{d} \rangle, \vec{b}_0)$ whenever the \mathcal{L} drops below some threshold.

3.4 Theoretical guarantees

Here we consider 2 theoretical aspects of our proposed solution method. We first note that FBA-POMCP converges to the optimal solution with respect to the belief, and secondly point out that the proposed belief tracking scheme converges to the true belief.

Analysis from [23] proof that the value function constructed by POMCP, given some suitable exploration constant u , converges to the optimal value function with respect to the initial belief. Work on BA-POMCP [11] extends the proofs to the BA-POMDP. Their proof relies on the fact that the BA-POMDP is a POMDP (that ultimately can be seen as a belief MDP), and that BA-POMCP simulates experiences with respect to the dynamics \bar{D} . These notions also apply to FBA-POMDP and we can directly extend the proofs to our solution method.

Given that FBA-POMCP converges to the optimal value function with respect to the belief, it is important to consider whether that the belief as a result of our particle reinvigoration approximates the true posterior (note that we are only concerned with the reinvigoration part of the belief update, as it is widely known that Importance Sampling with particle filters is unbiased). This follows directly from the convergence properties of Gibbs sampling, Metropolis-Hastings and MH-within-Gibbs that have been used to sample from the posterior. Since these methods are unbiased approximations and we use them directly to sample from the true

posterior $p(\langle s, G, \chi \rangle|\langle \vec{a}, \vec{d} \rangle, \vec{b}_0)$, we show that our solution method converges to the true distribution (given the initial belief).

4 EXPERIMENTS

Here we provide empirical support for our factored Bayes-Adaptive approach on 3 domains: the Factored Tiger, Collision Avoidance, and Gridworld problem. The Factored Tiger problem is an extension of the well-known Tiger problem [10], the Collision Avoidance problem is taken from [16] and the Gridworld is inspired by navigational tasks. In this section we first describe each domain on a high level, followed by the prior information we assume given to the agent. For more details please refer to the appendix.

4.1 Setup

The Tiger domain describes a scenario where the agent is faced with the task of opening one out of two doors. Behind one door lurks a tiger, a danger and reward of -100 that must be avoided, while the other door opens up to a bag of gold for a reward of 10. The agent can choose to open either doors (which ends the episode) or to listen for a signal: a noisy observation for a reward of -1 . This observation informs the agent of the location of the tiger with 85% accuracy. In the **Factored Tiger** domain we increase the state space artificially by adding 7 uninformative binary state features. While these features increase the state space, they are stationary over time and do not affect the observation function. From a planning point of view the problem retains its complexity regardless of the number of features, the challenge for a learning agent however is to infer the underlying dynamics in the significantly large domain.

In this particular case, the agent is unsure about the observation function. In particular, the prior belief of the agent assigns 60% probability to hearing the tiger correctly. The prior belief over the structure of the observation model is uniform. This means that each edge from any of the 8 state features to the observation feature has a 50% chance of being present in a particle in the initial belief.

In the **Collision Avoidance** problem the agent pilots a plane that flies from (centre figure) right to left (1 cell at a time) in a 2-dimensional grid. The agent can choose to stay level for no cost, or move either diagonally up or down with a reward of -1 . The episode ends when the plane reaches the column on the left, where it must avoid collision with a vertically moving obstacle (or face a reward of -1000). The obstacle movement is stochastic, and the agent observes its location at each time step with some noise. The optimal policy attempts to avoid the obstacle with as little vertical movement as possible.

While we assume the agent knows the observation and transition model of the plane, the agent initially underestimates the movement strategy of the obstacle: it believes it will stay put 90% of the time and move either direction with 5% probability each, while the actual probabilities are respectively 50% and 25%. The agent knows that the location of the obstacle in the next state depends on its previous location a-priori, but otherwise assume no initial acknowledge on the structure of the model with respect to the

location of the object.

Gridworld, is a 2-dimensional grid in which the agent starts in the bottom left corner and must navigate to a goal cell. The goal cell is chosen from a set of candidates at the start of an episode, and can be fully observed by the agent. The agent additionally observes its own location with a noisy localizer. The agent can move in all 4 directions, which are generally successful 95% of the attempts. There are, however, specific cells that significantly decrease the chance of success to 15%, essentially trapping the agent. The target of the agent is to reach the goal as fast as possible.

In this domain we assume no prior knowledge of the location or the number of ‘trap’ cells and the prior assigns 95% probability of transition success on all cells. The observation model in this domain is considered known. Here we factor the state space up into the location of the goal state and the (x, y) position of the agent and assume the agent knows that its next location is dependent on the previous, but is unsure whether the goal location is necessary to model its transition probabilities. This results in a prior belief where all particles contain models where, for each action, the x and y values of the current location are used to predict the next location of the agent, and half the particles *also* include the value of the goal cell as input edge.

4.2 Analysis

We compare **our method** to 3 other solution methods (fig. 1). We consider the **BA-POMCP** agent, as the baseline approach that ignores factorization and attempts to learn in the problems framed as BA-POMDPs. A second approach called **knows-structure** acts with complete prior knowledge of the structure of the dynamics and could be considered as a best-case scenario. This method requires additional knowledge of the domain and thus can be considered as ‘cheating’ compared to the other methods². Thirdly we test an agent **no-reinivation** with the same prior knowledge as our method, except that it does not reinvigorate its belief. The comparison with this approach highlights the contribution of the reinvigoration step proposed by us to keep a healthy distribution over the structure of the dynamics.

In order to produce statistically significant results we ran the experiments described above a large number of times. In these experiments, the parameters of the Monte-Carlo Tree Search planning and Importance Sampling belief update were equal across all solution methods. We refer to the appendix for more details. We first make the general observation that our method consistently either outperforms or is on par with the other methods, even compared to the knows-structure approach whose prior knowledge is more accurate.

In the Collision Avoidance domain our method outperforms even the agent with complete knowledge to the model topology with statistical significance knows-structure. This implies that particle reinvigoration (with respect to the true, complete posterior)

is beneficial even when structure degeneracy is not a problem, because it improves the belief over the model by better approximating the distribution over χ . The dynamics of this problem are particularly subtle, causing the belief over the graph topologies in the no-reinivation agent to converge to something different from the true model. As a result, it is unable to exploit the compactness of the true underlying dynamics and its performance is similar to BA-POMCP. None of the agents in the Collision Avoidance problem (left graph) have converged yet due to the lack of learning time provided in the 500 episodes (which we cut off in the interest of time).

Gridworld has comparatively less subtle transitions, and all methods show a generally quicker learning pace compared to other domains (centre figure). Nevertheless, BA-POMCP has not converged to the true model yet after 500 episodes whereas the factored approaches (in particular our method and knows-structure) do so after less than 200.

The results on the Factored Tiger problem (right figure) show significantly different behaviour. Firstly, the initial performance of both our method and no-reinivation is worse than the BA-POMCP and knows-structure. The reason becomes obvious once you realize that due to the uniform prior over the structure, half of the models in the initial belief contain topologies that cannot represent the intended prior counts. This leads to a change in the initial belief and thus the initial performance is different. With a uniform prior over the structure and without reinvigoration, the agent could accidentally end up converging to a model structure that is unable to express the true underlying dynamics. This is shown spectacularly by no-reinivation, where the average performance *reduces* over time. More detailed qualitative analysis showed that in most runs the agent actually performs similar to the other FBA-POMDP agents, however, every now and then the belief converges to a model structure that does not include the tiger location as parent feature in the observation model, leading to a policy that opens doors randomly with an expected return of -45 . Lastly, the lack of improvement of the BA-POMCP agent emphasises the need of factored representations most of all. Due to the large state space the number of variables in the observation model grows too large to learn individually. As a result, even 400 episodes are not enough for the agent to learn a model.

5 RELATED WORK

Much of the recent work in Reinforcement Learning in partially observable environments has been in applications of Deep Reinforcement Learning to POMDPs. To tackle the issue of remembering past observations, researchers have attempted to employ Recurrent networks [7, 26]. Others have introduced inductive biases into the network in order to learn a generative model to imitate belief updates [9]. While these approaches are able to tackle large-scale problems, they are not Bayesian and hence do not share the same theoretical guarantees.

More traditional approaches include the U-Tree algorithm [18] (and its modifications), EM-based algorithms such as [15] and policy

² Note, however that it is more common to know whether domain features share dependencies than it is to know the true probabilities

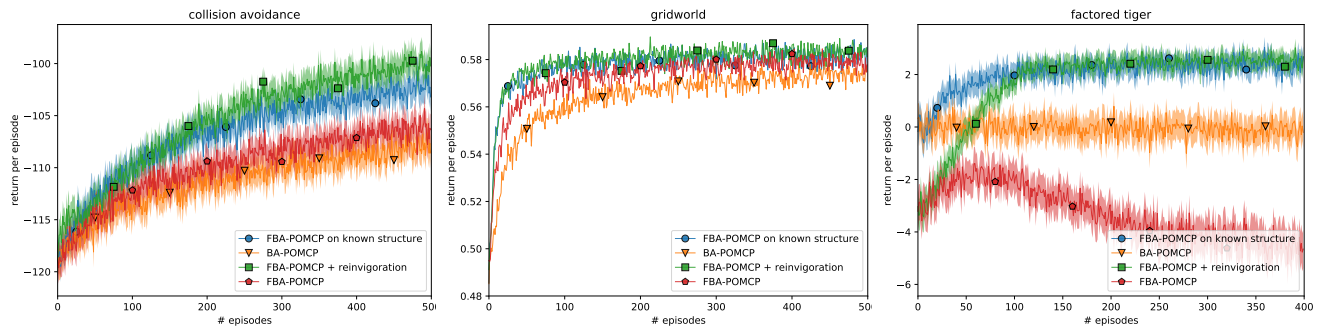


Figure 1: Average return on the Collision Avoidance (left), Gridworld (middle) and Factored Tiger (right) problem. The shaded areas indicate the 95% confidence interval.

gradient descent methods [2]. One of their main drawbacks is that they do not address the fundamental challenge of the exploration-exploitation trade-off in POMDPs.

There are other approaches that directly try to address this issue. The Infinite-RPR [14] is an example of a model-free approach. The Infinite-POMDP [5] is an example of a model-based solution method that attempts to do it in a model-based fashion as well. Their approach is similar in the sense that they learn a model in a Bayesian manner, however their assumptions of prior knowledge and about what is being learned are different.

6 CONCLUSION

This paper addresses the void for Bayesian Reinforcement Learning models and methods at the intersection of factored models and partially observable domains. Our approach describes the dynamics of the POMDP in terms of graphical models and allows the agent to maintain a joint belief over the state, and both the graph structure and CPT parameters simultaneously. Alongside the framework we introduced FBA-POMCP, a solution method, which consists of an extension of Monte-Carlo Tree Search to FBA-POMDPs, in addition to a particle reinvigorating belief tracking algorithm. The method is guaranteed to converge to the optimal policy with respect to the initial, as both the planner and the belief update are unbiased. Lastly we compared it to the current state-of-the-art approach on the 3 different domains. The results show the significance of representing and recognizing independent features, as our method either outperforms BA-POMDP based agents or is able to learn in scenarios where tabular methods are not feasible at all.

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