PRA1004 Scientific Computing 2013

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Lab Assignments Week 6

Overview Lab 6

In this lab we will cover...

- ... the implementation of Newton's method.
- ... Euler's method to 'simulate' ordinary differential equations, and comparing that to Matlab's build in functionality.

Since there is no next lab, you are required to show me this weeks work before the end of the lab to earn your mark.

1 Newton's Method: using a While loop(1h)

In this section you will implement Newton's method to compute the digits of $\sqrt{2}$.

- Write down the function f for which $\sqrt{2}$ is a root.
- Write down the derivative f'.
- Now the Newton method will perform iterations, computing a new estimate of the root as follows:

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$$

- Implement Newton's method to find the digits of $\sqrt{2}$. Use a while loop that continues iterations until the distance between the new and previous x is smaller than `eps(x)` (remember that `eps` gives the spacing around a point!)
- Use `format` to display all 16 significant digits.
- How many iterations does the algorithm need if you start with a crude approximation $x_0 = 4$?

2 Hand-in Assignment: Newton's Method for the Cliff Problem

Make a copy of your script that you made for section 1 and modify it such that it will solve a variant of our 'projectile' problem (illustrated in Figure 1):

> Anne is standing at the edge of a cliff of height h (m) and throws a rock with speed v (m/s) under an angle θ. We assume there is no friction, but that otherwise the experiment takes place on earth (so the gravitational acceleration $g = 9.8m/s^2$). How far does the rock travel horizontally before hitting the ground when $h = 14.5$m, $v = 30$m/s and $\theta = 34°$?

Hint: try first expressing both the horizontal and vertical position as a function of time, then find the time at which the rock hits the ground.

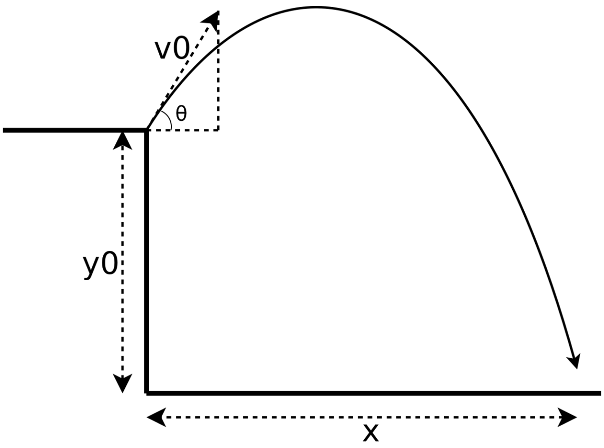

Figure 1: The cliff problem.

3 Differential Equations — Population Dynamics

The Euler method, described by Leonhard Euler around 1768, is the most basic approach to solving a ordinary differential equation with a given initial value.[1] Essentially it 'simulates' the differential equations using a particular step-size.

Use the forward Euler method to solve the following models for population dynamics. This means that you need to implement the updates in a `for` loop.

1. Unbounded exponential growth: $\frac{dp}{dt} = Cp$.
   (If you want to replicate the graph from the slides, use $p(0) = 12740, C = 0.1$, but chose anything you like.)
2. Capacity-bounded growth: $\frac{dp}{dt} = Cp\left(1 - \frac{p}{B}\right)$, where B is the capacity (the maximum population that can exist)).
3. Repeat the experiments, but now use Matlab's `ode45` function. What are the differences between it and your own forward Euler function?
   Note: Because this function makes use of the general $p' = f(t, p)$ formulation, this will require the definition of $\frac{dp}{dt}$ as a function with 2 arguments: 't' and 'p'! E.g. use `f1=@(t,p)[C*p]`.

[1] See also `http://en.wikipedia.org/wiki/Euler_method`.