PRA1004 Scientific Computing - Lab Assignments

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Week 6

Overview Lab 6

There is no new lab for week 6. However, for people that already finished the lab exercises, here are some suggestions to practice with the concepts of numerical differentiation / integration and simulation of differential equations.

1 Numerical Differentiation — Demography

Attempt "Exercise 4.4 (Demography)" from the text book.

2 Numerical Integration — Demography/Statistics

(See also Problem 4.4 on page 108 of QSG.)

The height of a population of people follows a bell-curve called *Gaussian, or normal distribution.* See, Figure 1 for an illustration. In particular it expresses the 'probability density' of height h. The formula is

$$p(h) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{h-\mu}{\sigma}\right)^2}$$

where

- h is the height for which we want to know the probability density.
- μ is the mean height (use 180).
- σ is the standard deviation (use 10).

(Also, note the difference with the formula in the book. The formula here is the standard normal distribution, which is a probability density function: its integral is 1. The book give a scaled version.)

Now, given this model, we can find the probability P that a randomly drawn person has height $h \in (190, 210)$ (or some other range) by taking the integral:

$$P = \int_{190}^{210} p(h) dh.$$

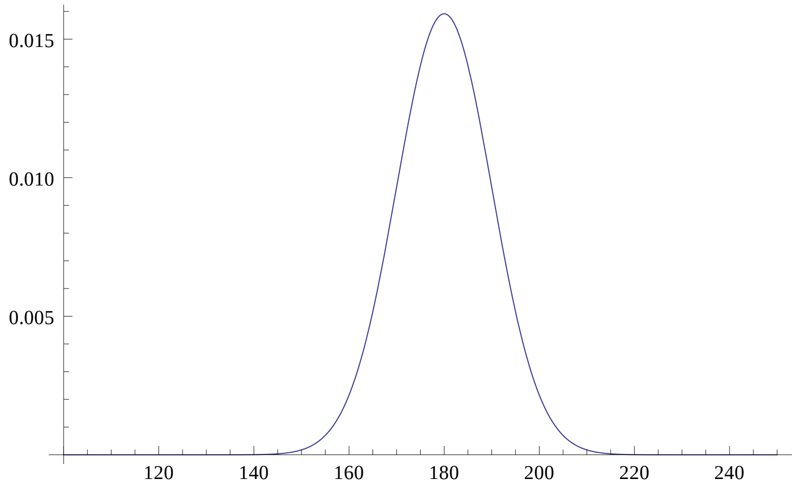

Figure 1: Used model of probability distribution of heights.

1. Approximate this integral using either the composite midpoint formula (4.13) or the trapezoid formula (from slides or (4.17) in QSG).
2. Check in Mathematica what the 'closed-form' solution is.
   Some pointers:
   - in Mathematica, all build in functions start with capital letters.
   - functions use square brackets.
   - use `p[h_] = (1/(sigma*2*Pi))*Exp[...]` to define the function $p(h)$.
   - use `Integrate[p[h], h]` to find the indefinite integral.
3. Also find in Mathematica the numerical approximation.
   (a) use `Integrate[p[h], {h,190,210}]` to find the definite integral.
   (b) use `N[...]` to find a numerical evaluation.
4. How close is your own solution to Mathematica's answer? Does decreasing the step size H get you closer to Mathematica's solution?

For the Mathematica part: save everything in the notebook (.nb file) and submit it with your report.

3 Differential Equations — Population Dynamics

Use the forward Euler method to solve ('simulate') the following models for population dynamics.

1. Unbounded exponential growth: $\frac{dp}{dt} = Cp$.
   (If you want to replicate the graph from the slides, use $p(0) = 12740, C = 0.1$, but chose anything you like.)

2. Capacity-bounded growth: $\frac{dp}{dt} = Cp\left(1 - \frac{p}{B}\right)$, where B is the capacity (the maximum population that can exist)).

3. Repeat the experiments, but now use Matlab's `ode45` function. What are the differences between it and your own forward Euler function?
   Note: Because this function makes use of the general $p' = f(t, p)$ formulation, this will require the definition of $\frac{dp}{dt}$ as a function with 2 arguments: 't' and 'p'! E.g. use `f1=@(t,p)[C*p]`.